

The $\phi \rightarrow \pi^+\pi^-$ decay within a chiral unitary approach

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Starting from the Chiral Perturbation Theory Lagrangian, but keeping different masses for the charged and neutral mesons ($m_u \neq m_d$), and using a previously developed non-perturbative unitary scheme that generates the lightest meson-meson resonances, we construct $K\bar{K} \rightarrow K\bar{K}$ and $K\bar{K} \rightarrow \pi^+\pi^-$ in the vector channel. This allows us to obtain the kaon-loop contribution to the $\phi - \rho$ mixing and study the $\phi \rightarrow \pi^+\pi^-$ decay. The dominant contribution to this decay comes from the $\phi \rightarrow \gamma \rightarrow \pi^+\pi^-$ process. However, there can be large interferences with the subdominant contributions coming from $\phi - \rho$ and $\phi - \omega$ mixing, or of these two contributions among themselves. As a consequence, a reliable measurement of $\phi \rightarrow \pi^+\pi^-$ decay could be used to differentiate between some $\phi - \omega$ mixing scenarios proposed in the literature.

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I. INTRODUCTION

The ϕ decay into $\pi^+\pi^-$ is an example of isospin violation, since the ϕ has isospin $I = 0$ and spin $J = 1$, and it would not couple to $\pi^+\pi^-$ in the isospin limit, which requires $I + J = \text{even}$ (the decay into $\pi^0\pi^0$ is forbidden in any case because the particles are identical). In addition, it violates the OZI rule [1] and hence it is subleading in the large N_c [2] expansion. The experimental situation on this decay is rather confusing. There are two old results: $BR = (1.94 + 1.03 - 0.81) \times 10^{-4}$ from [3], and $BR = (0.63 + 0.37 - 0.28) \times 10^{-4}$ [4], with very different central values but whose errors are so big to make them compatible. Very recently, two new, more precise, but conflicting results have been reported from the two experiments at the VEPP-2M in Novosibirsk: the CMD-2 Collaboration reports $BR = (2.20 \pm 0.25 \pm 0.20) \times 10^{-4}$ [5] whereas the SND Collaboration [6] obtains $BR = (0.71 \pm 0.11 \pm 0.09) \times 10^{-4}$.

On the theoretical side, the common ground is based on the $\phi - \rho$ mixing [7–9] to account for the strong part of the decay. In addition, in ref. [8] it has been pointed out

that the two-step $\phi - \omega - \rho$ transition* can give a relevant contribution and that other non-resonant processes, as a possible bare $\phi\rho\pi$ coupling, have to be considered in detail. It is remarkable, in contrast with the OZI allowed $\omega \rightarrow \pi^+\pi^-$ decay, that the electromagnetic $\phi\pi^+\pi^-$ coupling via photon exchange $\phi - \gamma - \rho - \pi^+\pi^-$ provides the right order of magnitude [7,9].

Within Chiral Perturbation Theory (ChPT) [10,11], isospin breaking has recently gained interest, since it is possible to take systematically into account the corrections due to the different u and d quark masses and to electromagnetic effects. Examples of such calculations are $\pi\pi$ scattering [12], some πN amplitudes and the nucleon self-energy [13], NN scattering [14] and the ponium atom [15].

Unfortunately, isospin violation in $\phi \rightarrow \pi^+\pi^-$ lies far away from the ChPT applicability range, since it involves the propa-

* As a matter of fact, this two-step process, just gives a contribution to the $\phi - \rho$ mixing. We will consider such resonant processes as the one that provides, by resonance saturation, the complementary local terms to the kaon loop contributions to $\phi - \rho$ mixing that we will calculate later on.

gation of the pair of mesons around 1 GeV. Nevertheless, new nonperturbative schemes imposing unitarity and still using the ChPT Lagrangian have emerged enlarging the convergence of the chiral expansion [16–18], for a review see ref. [19]. Here we shall follow the work [17], since it provides the most comprehensive study of the different meson-meson scattering channels, including resonances up to 1.2 GeV. In particular, this method yields a resonance in the $I = 0, J = 1$ channel, which is related to the ϕ and thus will allow us to obtain an important contribution to $\phi \rightarrow \pi^+\pi^-$ due to the charged and neutral meson mass difference. We shall also consider electromagnetic contributions at tree level as well as the contribution due to the $\phi - \omega$ mixing. These three contributions can have different kinds of cancellations among themselves, depending on the $\phi - \omega$ mixing scenario.

Some other theoretical uncertainties in our approach are unavoidable since the results are rather sensitive to the L_i coefficients of the $O(p^4)$ ChPT Lagrangian and to the value of F_V , which measures the coupling of a vector resonance with a photon. We will not calculate the electromagnetic loop corrections since the present ignorance of higher order counterterms makes their calculation unfeasible. However, from refs. [7,20,21] one expects the meson-photon intermediate states to yield a contribution of, at most, 25% of that of kaon loops.

II. TREE LEVEL CONTRIBUTIONS

A. The vector meson chiral Lagrangian

In order to calculate the contribution of an intermediate photon to $\phi \rightarrow \pi^+\pi^-$, we will use the vector meson chiral effective Lagrangian presented in [22], which is written in terms of the SU(3) pseudoscalar meson matrix ϕ and the antisymmetric vector tensor field $V_{\mu\nu}$ defined as

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix},$$

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}_{\mu\nu}. \quad (1)$$

The latter is normalized such that

$$\langle 0 | V_{\mu\nu} | P \rangle = \frac{i}{M_R} [P_\mu \epsilon_\nu(R) - P_\nu \epsilon_\mu(R)], \quad (2)$$

with M_R , P and $\epsilon_\mu(R)$ the mass, momentum, and polarization vector of the vector field R . Following [22], let us then consider the Lagrangian

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle, \quad (3)$$

where “ $\langle \rangle$ ” indicates the SU(3) trace and

$$u_\mu = i u^\dagger D_\mu U u^+ = u_\mu^\dagger$$

$$u^2 = U = \exp\left(\frac{i\sqrt{2}}{f}\phi\right)$$

$$D_\mu U = \partial_\mu U - i e [Q, U] A_\mu, \quad (4)$$

with Q the quark charge matrix

$$Q = \frac{1}{3} \text{diag}(2, -1, -1), \quad (5)$$

and A_μ the electromagnetic field. As usual, f is the pion decay constant in the chiral limit (we take $f \simeq f_\pi = 92.4$ MeV) and the $f_+^{\mu\nu}$ and $F^{\mu\nu}$ tensors are defined as

$$f_+^{\mu\nu} = u F^{\mu\nu} u^\dagger + u^\dagger F^{\mu\nu} u,$$

$$F^{\mu\nu} = e Q (\partial^\mu A^\nu - \partial^\nu A^\mu), \quad (6)$$

In order to introduce the ϕ and ω states, we extend SU(3) to U(3) and substitute

$$V_{\mu\nu} \rightarrow V_{\mu\nu} + (\omega_1)_{\mu\nu} \frac{I_3}{\sqrt{3}}, \quad (7)$$

where I_3 is the diagonal 3×3 matrix and ω_1 is the lightest singlet vector resonance. Hence, by imposing ideal mixing between ω_1 and ω_8

$$\frac{2}{\sqrt{6}}\omega_1 + \frac{1}{\sqrt{3}}\omega_8 = \omega^{ideal},$$

$$\frac{1}{\sqrt{3}}\omega_1 - \frac{2}{\sqrt{6}}\omega_8 = \phi^{ideal}, \quad (8)$$

Unless otherwise stated, in the following we will refer to these states simply as ω and ϕ , although it should be kept in mind that we are referring to their respective ideal states. Finally, the ϕ and ω can be introduced into the chiral notation by replacing in eq.(3) the $V_{\mu\nu}$ tensor by

$$\tilde{V}_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_{\mu\nu}. \quad (9)$$

The convention of signs of eq.(4) agrees with a more standard one if we take e negative in all the Lagrangians, as we shall do in what follows. The vertex function $\phi \rightarrow \gamma$, resulting from eq.(3), is

$$i \mathcal{L}_{\phi\gamma} \rightarrow -i \frac{\sqrt{2}}{3} |e| F_V M_\phi \epsilon^\mu(\phi) \epsilon_\mu(\gamma), \quad (10)$$

and to the same order than eq.(10) the Lagrangian giving the coupling of the photon to the pions is

$$i \mathcal{L}_{\gamma\pi^+\pi^-} = |e| (\pi^- \partial^\mu \pi^+ - \pi^+ \partial^\mu \pi^-) A_\mu \quad (11)$$

With these ingredients we can write the contribution of the Feynman diagram of fig.1, which is given by

$$ie^2 \frac{\sqrt{2} F_V}{3 M_\phi} \epsilon^\mu(\phi) (p_{\pi^+} - p_{\pi^-})_\mu F(M_\phi^2), \quad (12)$$

where $F(q^2)$ is the pion electromagnetic form factor, which at the ϕ mass is given by $F(M_\phi^2) = -1.56 + i 0.66$ [23].

This can be compared with the coupling of the ϕ to $K^+ K^-$, or $K^0 \bar{K}^0$, which can be obtained from the G_V term in eq.(3) and reads

$$i \mathcal{L}_{\phi K^+ K^-} \rightarrow -i g_{\phi K^+ K^-} \epsilon^\mu(\phi) (p_{K^+} - p_{K^-})_\mu, \quad (13)$$

$$g_{\phi K^+ K^-} = \frac{s G_V}{\sqrt{2} f^2 M_\phi}.$$

The $\phi \rightarrow K^+ K^-$ width is then given by

$$\Gamma_{\phi K^+ K^-} = \frac{p_{K^+}^3}{6\pi M_\phi^2} g_{\phi K^+ K^-}^2, \quad (14)$$

which, using its experimental value [24], provides $G_V = 54.3$ MeV (to compare with $G_V = 53$ MeV, from the study of the pion EM radius [11,22]).

By analogy to eq.(13), eq.(12) provides a $\phi\pi^+\pi^-$ coupling

$$g_{\phi\pi^+\pi^-}^{(\gamma)} = -\frac{\sqrt{2}}{3} e^2 \frac{F_V}{M_\phi} F(M_\phi^2), \quad (15)$$

and eq.(14), substituting $g_{\phi K^+ K^-}$ by $g_{\phi\pi^+\pi^-}$ and p_{K^+} by p_{π^+} , provides the tree level electromagnetic contribution to the $\phi \rightarrow \pi^+\pi^-$ decay width. With a value of $F_V = 154$ MeV from the $\rho \rightarrow e^+e^-$ decay [22] this contribution alone would yield $BR(\phi \rightarrow \pi^+\pi^-) = 1.7 \times 10^{-4}$, a value compatible with the experiment of ref. [5], within errors.

B. Comparison with $\omega \rightarrow \pi^+\pi^-$

It may seem surprising that $g_{\phi\pi^+\pi^-}^{(\gamma)}$ already provides the correct order of magnitude of the $\phi \rightarrow \pi^+\pi^-$ decay, since, in contrast, it is well known [25,26] that the tree level photon contribution $\omega \rightarrow \gamma \rightarrow \rho \rightarrow \pi^+\pi^-$, represents a negligible amount of the $\Gamma(\omega \rightarrow \pi^+\pi^-)$.

However, the case of the ω is radically different from ours and can be well understood from $\rho - \omega$ mixing. We will now calculate this effect making use of an effective chiral Lagrangian and large N_c arguments [27]. Indeed, from this reference, the $\rho - \omega$ mixing can be represented as

$$i \mathcal{L}_{\rho\omega} \rightarrow i \tilde{\Theta}_{\rho\omega} \epsilon_\rho \cdot \epsilon_\omega, \quad (16)$$

with

$$\tilde{\Theta}_{\rho\omega} = \frac{s}{M_V^2} \left[-(m_{K^0}^2 - m_{K^+}^2) + (m_{\pi^0}^2 - m_{\pi^+}^2) + \frac{1}{3} F_V^2 e^2 \right]. \quad (17)$$

Note that M_V is the mass of the vector octet in the chiral limit $M_V \approx M_\rho$ [22]. We have also made use in eq. (16) of eq. (2), thus turning to the usual vector notation. At lowest order in ChPT [11] the first two terms in eq. (17) arise from the quark mass difference, and the third one is of electromagnetic origin from the exchange of a photon between the ρ and the ω . It is straightforward to see that the electromagnetic contribution only amounts to a 14% of that due to quark mass differences.

Contrary to the ϕ case, the $\rho - \omega$ mixing is OZI allowed and leading in large N_c , as can be seen from eqs. (16) and (17). In fact, this term is of the same order than the free Lagrangian, both in the $1/N_c$ and in chiral countings (this is more clearly seen in the tensor notation).

In addition, there is a kaon loop contribution, fig.2, which, from ChPT, is expected

to be of the same order of magnitude than the electromagnetic contribution. Evaluating the diagram of fig.2 one has:

$$\tilde{\Theta}_{\rho\omega}^{Kaon-loops} = \frac{G_V^2}{f_\pi^4} \frac{s^2}{(4\pi)^2 M_V^2} [L(s, m_{K^+}) - L(s, m_{K^0})] \quad (18)$$

where, once again, we have used eq. (2) to present our results in the vector notation. The $L(s, m)$ loop function, in the usual ChPT $\overline{MS} - 1$ scheme, is:

$$L(s, m) = \frac{m^2 - s/6}{3} - \frac{m^2}{6} \log \frac{m^2}{\mu^2} - \frac{s - 4m^2}{12} \left[1 - \log \frac{m^2}{\mu^2} - \sigma \log \frac{\sigma + 1}{\sigma - 1} \right] \quad (19)$$

$$\sigma = \sqrt{1 - \frac{4m^2}{s}},$$

where μ is the dimensional regularization scale. In order to estimate the eq. (18) contribution at $s = M_\rho^2$, we use the natural value $\mu = \Lambda_{\text{ChPT}} \approx M_\rho$. The results depend on the regularization scale but they provide a good estimate of the order of magnitude, as we shall see later on, when we will reevaluate this contribution within the chiral unitary approach.

At this point we are ready to compare all contributions:

Quark mass differences from eq.(17)	= -5221.6 MeV ²	We have already obtained $\tilde{\Theta}_{\rho\omega}$, although
EM contribution from eq.(17)	= 725.1 MeV ²	has to be evaluated at $\sqrt{s} = M_\phi$.
Kaon loops from eq.(18)	= -136.5 MeV ²	The dominant contribution comes from

Hence, the $\rho - \omega$ mixing is dominated by the OZI allowed strong contribution due to quark mass differences, which is leading both in the large N_c and chiral countings. In addition, the kaon loops are smaller than the electromagnetic contribution although with a large destructive interference between them (for $G_V = 65$ MeV, which is the value needed to reproduce $\Gamma(\rho \rightarrow \pi^+\pi^-)$ from eq. (3), the estimate of the kaon loop contribution would be -190 MeV²). We will find again this large destructive interference between the kaon loops and the electromagnetic contribution when considering the ϕ resonance.

In summary, the fact that the purely electromagnetic contribution already provides a reasonable order of magnitude for the $\phi \rightarrow \pi^+\pi^-$ decay, is due to the absence of the OZI allowed contribution, which makes

the $\omega \rightarrow \pi^+\pi^-$ decay comparatively much larger. Note that such contribution is missing in the OZI violating, large N_c subleading, $\phi - \rho$ mixing. The fact that $\omega \rightarrow \pi^+\pi^-$ is much larger than the $\phi \rightarrow \pi^+\pi^-$ dominant contribution is very relevant since, through the $\phi - \omega$ mixing, it provides an additional mechanism that has to be taken into account in the complete calculation of $\phi \rightarrow \pi^+\pi^-$, and that we analyze next.

C. The “two step” $\phi - \omega - \rho$ mechanism

As a matter of fact, the physical ϕ and ω states are not the ideal ones defined in eq.(8), but instead

$$\begin{aligned} \omega &\simeq \omega^{ideal} - \delta_V \phi^{ideal} \\ \phi &\simeq \delta_V \omega^{ideal} + \phi^{ideal} \end{aligned}$$

In the literature there is a general agreement on $|\delta_V| \simeq 0.05$, but, apart from conventions, not on its sign [8]. Its contribution to the $\phi\pi^+\pi^-$ effective coupling is obtained from fig.3 as follows:

$$g_{\phi\pi^+\pi^-}^{(\phi\omega)} = -\frac{M_\phi^2 G_\rho}{M_\rho f^2} \frac{\tilde{\Theta}_{\rho\omega}(M_\phi)}{M_\phi^2 - M_\rho^2 + iM_\rho \Gamma_\rho} \frac{\tilde{\Theta}_{\phi\omega}(M_\phi)}{M_\phi^2 - M_\omega^2 + iM_\omega \Gamma_\omega}. \quad (20)$$

We have already obtained $\tilde{\Theta}_{\rho\omega}$, although the EM contribution has to be evaluated at $\sqrt{s} = M_\phi$. Since the dominant contribution comes from the quark mass differences. The kaon loop contribution cannot be calculated using eq.(18), since that formula is not unitary. We will see later, how this number can be obtained from the chiral unitary approach, and again it is of the order of 200 MeV², and therefore numerically irrelevant for the following discussion.

The new $\tilde{\Theta}_{\phi\omega}$ parameter can be obtained from the literature. Nevertheless, its imaginary part can be obtained from unitarity. The most relevant intermediate states are $K\bar{K}$ and three pions. In the first case the couplings to ϕ and ω are completely determined by the vector resonance Lagrangian. However, the imaginary part contribution of three pion intermediate states has some model dependence [8], mostly through the $g_{\phi\rho\pi}$ coupling.

We consider now two different scenarios for the $\phi - \omega$ mixing which illustrates to some

extent the uncertainties that are found in the literature with respect to this issue:

- “Weak mixing” scenario [8], where $\text{Re } \tilde{\Theta}_{\phi\omega} = 0$ and $g_{\phi\rho\pi} = 0.78 \text{ GeV}^{-1}$.
- “Strong mixing” scenario [8], where $\text{Re } \tilde{\Theta}_{\phi\omega} = 20000$ to 29000 MeV^2 and $g_{\phi\rho\pi} = 0$.

which will therefore appear as different cases in our final result.

Up to now we have just concentrated on the tree level diagrams of the $\phi\pi^+\pi^-$ decay. There are, however, important contributions from kaon loops that we will analyze in the next sections, whose calculation is the main novelty of this work.

III. DIRECT KAON LOOP CONTRIBUTION TO $\phi - \rho$ MIXING

A. Introduction

The pure strong interaction chiral Lagrangian gives a contribution to the $\phi \rightarrow \pi^+\pi^-$ decay if the charged and neutral meson masses are different, otherwise it would be forbidden.

For instance, from eqs.(3) and (9) there is no direct $\phi\pi^+\pi^-$ coupling. However, we can generate a non vanishing $\phi \rightarrow \pi^+\pi^-$ transition when keeping different masses for the charged and neutral kaons in the loops of fig.4, which do not violate the OZI rule, although they are subleading in large N_c . In fact, these diagrams are expected to give the main strong interaction contribution to $\phi \rightarrow \pi^+\pi^-$ due to intermediate states. For instance, the ϕ couples much more strongly to $K\bar{K}$ than to 3π , as it is clear from the fact that $\Gamma(\phi \rightarrow 3\pi)/\Gamma(K\bar{K}) \approx 1/5$, although three pions are kinematically much more favored than two kaons [†].

Note that in the evaluation of the diagrams of fig.4 the $K\bar{K} \rightarrow \pi^+\pi^-$ amplitude can receive important contributions from the ω or ρ exchange. In the first case, the

ω couples to the ρ once again, and therefore is included in the $\phi - \omega - \rho$ mixing contributions. Thus, in the following we will concentrate on the evaluation of these kaon-loop contributions to the direct $\phi - \rho$ mixing, that is, we will consider only the exchange of the ρ in the $K\bar{K} \rightarrow \pi^+\pi^-$ $I = 0$ P-wave amplitudes appearing on fig.4.

An estimation of the imaginary part of this contribution to the diagrams in fig.4 is straightforward using the vector meson chiral Lagrangian. The sum of the diagrams does not vanish due to the different masses of the charged and neutral kaons. The $\phi \rightarrow \pi^+\pi^-$ branching ratio that would be obtained taking into account just this contribution is already of the order of magnitude of the experimental results, given the large uncertainty on the data.

Yet, this estimate does not take into account corrections to $K\bar{K} \rightarrow \pi^+\pi^-$ due to isospin violation. In addition, the real part of the loop remains ambiguous since it requires the knowledge of higher order contributions than those given by eq.(3), that is, counterterms to absorb loop divergences. Furthermore, even when we have such counterterms, the chiral expansion is only expected to work at energies which are below the ϕ mass.

B. Resonances and the IAM

We present here a method which deals simultaneously with all these problems in order to extract the aforementioned kaon loop contributions. The method exploits the information of ChPT up to $\mathcal{O}(p^4)$, by relying on the expansion of the T^{-1} -matrix. The technique starts from the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ ChPT Lagrangian and uses the inverse amplitude method (IAM) in coupled channels. Unitarity provides for free the imaginary part of T^{-1} , and then a chiral expansion is done for $\text{Re } T^{-1}$, which, in the present case, has a larger radius of convergence than T itself. This approach has been applied in the isospin limit with remarkable results: with just one channel [16] it nicely describes the σ , ρ and K^* regions, amongst others, in $\pi^+\pi^-$ and πK scattering. When generalized to coupled channels [17,18] it also describes meson-meson scattering with all the associated resonances up to about 1.2 GeV. A more general approach is used in [28] by

[†]We will address in subsection III.D the problem raised in refs. [47,48], relative to the contributions of more massive virtual intermediate states

means of the N/D method, in order to include the exchange of some preexisting resonances explicitly, which are then responsible for the values of the fourth order chiral parameters.

The T amplitude is defined in terms of the partial waves as

$$T = \sum_J (2J+1) T_J(s) P_J(\cos \theta), \quad (21)$$

In what follows we will refer to T_J simply as T . Within the coupled channel formalism, the IAM partial wave amplitude is given by the matrix equation

$$T = T_2 [T_2 - T_4]^{-1} T_2, \quad (22)$$

where T_2 and T_4 are $O(p^2)$ and $O(p^4)$ ChPT partial waves, respectively. In principle, T_4 would require a full one-loop calculation, but it was shown in [17] that, at the phenomenological level, it can be well approximated by

$$\text{Re } T_4 \simeq T_4^P + T_2 \text{Re } G T_2 \quad (23)$$

where T_4^P is the tree level polynomial contribution coming from the \mathcal{L}_4 chiral Lagrangian and G is a diagonal matrix $\text{diag}(g_1, g_2, g_3)$, where g_i is the loop function of the intermediate two meson propagators, which we give in the appendix. In [17] the loop integrals are regularized by means of a momentum cut-off, q_{max} , in the loop three-momentum. The relation between this cut-off and the dimensional regularization scale μ , normally used in ChPT, is also given in that paper.

We have also taken advantage to correct a small error detected in [17] in the $K^+ K^- \rightarrow K^0 \bar{K}^0$ amplitude, whose complete expression in the isospin limit is given in the appendix. We have also reconduted a fit to the data including those on $(\delta_{00} - \delta_{11})$, which are well determined from [29]. The fit of the phase shifts and inelasticities is carried out here in the isospin limit, as done in [17]. There are several sets of L_i coefficients which give rise to equally acceptable fits.

As it can be seen in fig.5, there are several plots for which there are incompatible sets of data. This is particularly evident for the δ_{00} data both in $\pi\pi \rightarrow \pi\pi$ and $K\bar{K}$, in the inelasticity η_{00} , and in the $\delta_{01/2}$ phase shifts. As a consequence, although we have performed a χ^2 fit of the data using MINUIT [30], the resulting χ^2 per degree of freedom is

not really very meaningful, since the L_i values depend on the estimate of the systematic error of each experiment, which is not given in many original references. In addition, due to the fact that we have eight parameters, there are several χ^2 minima, which yield very similar values of χ^2 for rather different values of some chiral parameters. Which one is the real minimum depends on how we add the systematics. For that reason we have preferred to give several sets of coefficients, which yield $\chi^2/d.o.f < 2$ when assuming a 3% systematic error added in quadrature to the statistical error quoted by each experiment.

We write in table 1 the different sets of chiral parameters and we show their corresponding results for the phase shifts and inelasticities in fig.5. We can see that the small differences in the results appear basically only in the $a_0(980)$ and $\kappa(900)$ resonance regions, where data have also larger errors or are very scarce.

Although the tadpoles and loop terms in the crossed channels were neglected and reabsorbed into L_i redefinitions [17] when we use eq.(23), these coefficients are still close to those of standard ChPT (see Table I). Consequently, it seems that this simplifying approximation has a small effect in the relevant energy region, not spoiling also the standard low energy ChPT results.

One of the side consequences of the approach was the generation of a resonance around 1 GeV in the $I = 0$ and $J = 1$ channel, which only couples to $K\bar{K}$. Actually, it has a zero width, since its mass is below the $K\bar{K}$ threshold. One is tempted to associate this state to the ϕ meson, however, we can only relate it with the octet part ω_8 , which, by mixing with a singlet generates the ϕ and the ω . This can be easily understood since the singlet in this channel, ω_1 , which is symmetric in the SU(3) representation, does not couple to two mesons because their spatial wave function is antisymmetric. Since only two meson states were considered in [17,18], ω_1 does not appear in the IAM, and the resonant state found in that channel can only be related to ω_8 . However, we will see next that we can still exploit the properties of the w_8 pole in order to study the decays of the ϕ resonance.

C. Extracting the $\phi\pi^+\pi^-$ coupling from the IAM

Let us then turn to the case of interest for this work: the evaluation of the $J = 1$ $K\bar{K} \rightarrow K\bar{K}$ and $K\bar{K} \rightarrow \pi^+\pi^-$ amplitudes around the mass of the ω_8 . Now we are breaking isospin explicitly by keeping different the charged and neutral meson masses, while keeping the L_i obtained from the previous fits to meson-meson scattering in the isospin limit. In addition, we are dealing with three two-meson states: K^+K^- , $K^0\bar{K}^0$ and $\pi^+\pi^-$, that we will call 1, 2 and 3, respectively. The amplitude is a 3×3 matrix whose elements we will denote as T_{ij} (for instance, T_{13} stands for the $J = 1$ $K^+K^- \rightarrow \pi^+\pi^-$ amplitude). The T_2 and T_4^P amplitudes used in the present work and calculated in the isospin breaking case, are collected in the appendix.

Once the amplitudes are unitarized with the IAM, one observes the presence of two poles, one corresponding to the $\rho(770)$ and the other one to the w_8 resonance. It is interesting to note that the w_8 pole appears with a mass around 910 MeV, very close to the value 930 MeV predicted by the quadratic or linear $SU(3)$ mass formulae [46] for the w_8 mass. In the following, we will denote by Ω_8 the resonance pole that we have obtained in our approach corresponding to the w_8 resonance. The motivation for this change of notation is the lack of the 3π state in our model since this contribution can be particularly relevant in order to study certain properties of the w_8 resonance. For instance, the 3π couplings of the w_8 and w_1 according to eq. (8) add for the ω giving rise to the $w \rightarrow 3\pi$ coupling and almost cancel each other in the case of the ϕ , $|g_{\phi \rightarrow 3\pi}| \ll |g_{\omega \rightarrow 3\pi}|$.

In order to evaluate the kaon loop contribution to the $\phi\pi^+\pi^-$ coupling via direct $\phi - \rho$ mixing, we first study the $\Omega_8\pi^+\pi^-$ coupling. We thus evaluate the $K^+K^- \rightarrow K^+K^-$ amplitude (T_{11}) and the $K^+K^- \rightarrow \pi^+\pi^-$ amplitude (T_{13}) near the pole of the Ω_8 resonance. Close to the Ω_8 pole the amplitudes obtained numerically are then dominated by the exchange of this resonance, represented diagrammatically in fig 6.

By considering couplings like those in eq.(13) for the Ω_8 to K^+K^- and $\pi^+\pi^-$, these two amplitudes, once projected in the $J = 1$ channel, and close to the Ω_8 pole, are given by

$$T_{11} = g_{\Omega_8 K^+ K^-}^2 \frac{1}{s - M_{\Omega_8}^2} \frac{4 p_K p_{K'}}{3}$$

$$T_{13} = g_{\Omega_8 K^+ K^-} g_{\Omega_8 \pi^+ \pi^-} \frac{1}{s - M_{\Omega_8}^2} \frac{4 p_K p_\pi}{3}. \quad (24)$$

where p_i is the modulus of the center of mass three-momentum of the i particle. The diagram of fig.6b can be interpreted as providing an effective strong $g_{\Omega_8 \pi^+ \pi^-}^{(s)}$ coupling.

By looking at the residues of the T_{11} and T_{13} amplitudes in the Ω_8 pole we can get $g_{\Omega_8 K^+ K^-}$, $g_{\Omega_8 K^0 \bar{K}^0}$ and $g_{\Omega_8 K^+ K^-} g_{\Omega_8 \pi^+ \pi^-}$. Thus, defining

$$Q_{ij} = \lim_{s \rightarrow M_{\Omega_8}^2} (s - M_{\Omega_8}^2) \frac{3 T_{ij}}{4 p_i p_j} \quad (25)$$

we obtain

$$\frac{g_{\Omega_8 \pi^+ \pi^-}}{g_{\Omega_8 K^+ K^-}} = \frac{Q_{13}}{Q_{11}}. \quad (26)$$

In eq.(25) the T_{11}, T_{13} amplitudes have a large ρ exchange background, which can be eliminated using the residue of the Ω_8 pole obtained via eq.(25). Yet, numerically this background can be eliminated to a large extent by using the isospin zero combination $-(K^+K^- + K^0\bar{K}^0)/\sqrt{2}$ in the initial state. Hence, the $g_{\Omega_8 \pi^+ \pi^-}$ is more efficiently evaluated by means of the combination

$$\frac{g_{\Omega_8 \pi^+ \pi^-}}{g_{\Omega_8 K^+ K^-}} = \frac{Q_{13} + Q_{23}}{Q_{11} + Q_{21}}. \quad (27)$$

We have checked numerically that the $g_{\Omega_8 K^+ K^-}$ and the $g_{\Omega_8 K^0 \bar{K}^0}$ couplings have the same value in our approach. Since we are interested in ϕ , we still have to make the connection between the $\Omega_8\pi^+\pi^-$ and $\phi\pi^+\pi^-$ couplings. Indeed, we have explicitly checked that, when removing the rescattering resummation implicit in the IAM (by setting $G = 0$, see eq. (23)), the ratio in eq. (27) becomes between one and two orders of magnitude smaller. Even more, this drastic reduction in the $\Omega_8\pi^+\pi^-$ coupling is also obtained when making $G = \text{diag}(0, 0, g_3)$, that is, when only removing the kaon loops. Therefore the Ω_8 decays to $\pi^+\pi^-$ mainly through the mechanism shown in fig.4 (replacing the ϕ by the Ω_8). This observation allows us to find the kaon loop contribution to $\phi \rightarrow \pi^+\pi^-$ that we are looking for, through the same mechanisms of the Ω_8 , since the only difference will be the initial $\Omega_8 K\bar{K}$ and $\phi K\bar{K}$ couplings, which can be canceled taking the following ratio

$$\frac{g_{\phi\pi^+\pi^-}^{(s)}}{g_{\phi K^+K^-}} = \frac{g_{\Omega_8\pi^+\pi^-}}{g_{\Omega_8 K^+K^-}}. \quad (28)$$

Therefore, from eq. (26), one has

$$g_{\phi\pi^+\pi^-}^{(s)}(s) = \frac{Q_{13} + Q_{23}}{Q_{11} + Q_{21}} g_{\phi K^+K^-}(s), \quad (29)$$

with $g_{\phi K^+K^-}$ given in eq. (13). Here we are neglecting the mass difference between the Ω_8 and the ϕ resonance, which is around 100 MeV. In any case one has to take into account that: 1) The important ρ exchange effect is also canceled in the ratios. 2) We have removed in eq. (25) the three-momenta factors. As a result, the remaining differences coming from the mass difference should be rather tiny.

Finally, by adding the above contribution with that of eq.(15), we find

$$g_{\phi\pi^+\pi^-} = g_{\phi\pi^+\pi^-}^{(\gamma)} + g_{\phi\pi^+\pi^-}^{(\phi\omega)} + g_{\phi\pi^+\pi^-}^{(s)}, \quad (30)$$

which allows us to obtain the $\phi \rightarrow \pi^+\pi^-$ decay width as we did before only for the $g_{\phi\pi^+\pi^-}^{(\gamma)}$ coupling. In order to determine the sign of the interference in eq.(30) it is important to know the sign of $F_V G_V$ (see eqs.(13) and (12)). We have taken $F_V G_V > 0$ since the L_9 chiral parameter, whose main resonance contribution is given by $F_V G_V/2M_\rho^2$ [22], is positive and large.

D. The OZI rule violation

The direct coupling $g_{\phi\pi^+\pi^-}$ violates the OZI rule. This is clearly seen in a quark picture when considering the ϕ as a pure $s\bar{s}$ state. From the QCD Lagrangian one can see the OZI rule as a prediction of the $1/N_c$ expansion, with N_c the number of colors. While the couplings of the decays which do not violate the OZI rule are $O(1/N_c^{1/2})$ [2], those that violate the OZI rule are suppressed by an extra $1/N_c$. In addition, meson loops are suppressed by at least one power of $1/N_c$ [2]. As a consequence, the $g_{\phi\pi^+\pi^-}^{(s)}$ coupling given in eq. (29), which is due to kaon loops, as discussed above, is $O(1/N_c^{3/2})$. Note, in contrast, that the $g_{\phi K^+K^-}$ coupling, from eq. (13), is order $1/N_c^{1/2}$, since f and G_V are $O(N_c^{1/2})$ and M_ϕ is order 1.

However, in quark model calculations [47] the large N_c suppression of two intermediate meson states is considered insufficient in order to explain the experimental success of the OZI rule. The point is that in these models the real parts of the two meson loop contributions to OZI violating processes, although large N_c subleading, are found to be much larger than they should be in order to explain the experimental success of the OZI rule. The solution advocated by the authors is that a cancellation among a very large number of intermediate states seems to operate. This is illustrated via the example of $\omega - \rho$ mixing in [47].

Nevertheless, one should notice that the real part of the two-meson loop is divergent, and the remnant finite part depends upon the regularization and renormalization schemes, apart, of course, from the details of the dynamical model. In refs. [47,48] this regularization is done including several cut-offs within a quark flux tube model, having an explicit scale dependence. In contrast, we have just included kaons and pions as intermediate states and we have renormalized such contributions making use of a cut-off $\approx \Lambda_{\text{ChPT}}$. Still, the physical quantities we calculate are scale independent and well defined, since any change in the cut-off would be reabsorbed by a change in the L_i ChPT counterterms. Note that, since we are making use of an effective field theory formalism, the chiral Lagrangian counterterms should take into account any other contribution from more massive intermediate states. In our approach we use ChPT up to $O(p^4)$ and generate higher orders through eq. (22). In this way, any other contribution coming from heavier virtual intermediate states is reabsorbed in the final values of the L_i counterterms given in table 1. At this point, our previous statement about the fact that our result for the $g_{\phi\pi^+\pi^-}^{(s)}$ coupling is due to kaon loops is meaningful only because we have taken a natural value for the cut-off. For such value, the contribution from graphs without kaon or pion loops, which come just from the L_i counterterms, is between one and two orders of magnitude smaller than that of kaon loops. Comparing our work with that of refs. [47,48], we cannot tell exactly the size of each separate contribution due to each state more massive than the kaons. If each one of these contributions was large as it happens in refs. [47,48], then,

we would also be finding a cancellation.

In order to obtain further support for our arguments about the kaon loop size, it is instructive to revisit, within the IAM formalism, the kaon loop contribution to $\omega \rightarrow \pi^+\pi^-$ that we estimated in sect.II.B. Note that the value obtained for the $\omega - \rho$ mixing from kaon loops in sect.II.B was dependent on the regularization scale. In contrast, in the IAM this dependence is canceled with that of the chiral parameters L_i . In addition, the IAM respects unitarity and accounts for isospin breaking not only in the loops (through different masses of the charged and neutral kaons), but also in the ϕK^+K^- and $\phi K^0\bar{K}^0$ couplings and the $K\bar{K} \rightarrow \pi^+\pi^-$ amplitudes.

In order to reinterpret our results for the $\Omega_8\pi^+\pi^-$ coupling in terms of an $\Omega_8 - \rho$ mixing and compare with sect.II.A., we write (see fig.7)

$$g_{\Omega_8\pi^+\pi^-} = \tilde{\Theta}_{\Omega_8\rho} g_{\rho\pi^+\pi^-} \frac{1}{M_{\Omega_8}^2 - M_\rho^2 + i M_\rho \Gamma_\rho}, \quad (31)$$

with $g_{\rho\pi^+\pi^-} = -G_V s / (f^2 M_\rho^2)$ from eq.(3). This gives us $\tilde{\Theta}_{\Omega_8\rho}$, from where, using eq.(8) and the fact that the ω_1 does not couple to $K\bar{K}$ at the leading chiral order, we obtain

$$\tilde{\Theta}_{\omega\rho} = \frac{1}{\sqrt{3}} \tilde{\Theta}_{\Omega_8\rho} = \frac{1}{\sqrt{3}} \frac{g_{\Omega_8\pi^+\pi^-}}{g_{\rho\pi^+\pi^-}} [M_{\Omega_8}^2 - M_\rho^2 + i M_\rho \Gamma_\rho] \mathbf{A}. \quad \phi \rightarrow \pi^+\pi^- \quad \text{Branching Ratio} \quad (32)$$

Taking now the value for $g_{\Omega_8\pi^+\pi^-}$ obtained in the IAM from eq.(27), with $g_{\Omega_8 K^+K^-} = -\sqrt{3/2} g_{\phi K^+K^-}$ from eq.(3), we arrive at a value of $\tilde{\Theta}_{\omega\rho}(M_\rho) = (-52 - i76) \text{ MeV}^2$ and $\tilde{\Theta}_{\omega\rho}(M_\phi) = (-299 - i81) \text{ MeV}^2$. These results corroborate the “order of magnitude” arguments given in sect.II.B., obtained using the non-unitary eq.(18), to show that the kaon loop contributions are very small relative to the dominant OZI allowed contribution.

It is also interesting to remark that the cancellation between mesons loops in the model of ref. [47] does not operate for the scalar sector with vacuum quantum numbers $J^{PC} = 0^{++}$ as discussed in ref. [48]. The failure of the large N_c suppression in this sector, and its associated OZI rule violation, is also discussed in more general terms in ref. [49]. Although the scalar

sector is very hard to discuss in terms of quark models, due to the large rescattering effects, it is equally well described as the vector channels in the framework of non-perturbative unitarity methods from the ChPT series [17,18,28,50,51], see also fig. 5. For instance, in refs. [50,51] the σ , $f_0(980)$ and $a_0(980)$ were dynamically generated and their meson-meson and $\gamma\gamma$ decay modes were analyzed in very good agreement with experiment. Furthermore, in ref. [28] the spectrum in the scalar sector was discussed taking into account as well the large N_c limit. In addition, the presence of a scalar nonet due to the meson-meson self interactions, which disappears in the limit $N_c \rightarrow \infty$, was then established. On the other hand, it was also found that the lightest preexisting scalar nonet, with mass $\mathcal{O}(1)$ in the N_c counting, should comprise a singlet around 1 GeV and an octet around 1.4 GeV, in qualitative agreement with the expectations of ref. [48]. The success of our approach in the 0^{++} sector indicates that our techniques are powerful in the study of OZI violating processes. Note that we describe both vector and scalar channels without including any new ad-hoc elements.

IV. RESULTS AND DISCUSSION

In this section we are going to present the resulting branching ratios for the $\phi \rightarrow \pi^+\pi^-$ decay. To do that we will consider and discuss the different sources contributing to the total $g_{\phi\pi^+\pi^-}$ coupling as given in eq.(30).

We first consider the contribution $g_{\phi\pi^+\pi^-}^{(\gamma)}$ introduced in sect.II.A. We take as a final value $g_{\phi\pi^+\pi^-}^{(\gamma)} \simeq [10.6 \pm 0.4 - i(4.47 \pm 0.15)]10^{-3}$ where the uncertainty is mainly due to the value of F_V , which ranges between $F_V = 154 \text{ MeV}$, coming from the $\rho \rightarrow e^+e^-$ decay, and $F_V = 165 \text{ MeV}$, coming from the $\phi \rightarrow e^+e^-$ decay, when evaluating both them with eqs. (3) and (9).

Concerning the kaon-loop contributions to the $\phi - \rho$ mixing eq.(29), after averaging over all the fits presented in table 1, we obtain:

$$g_{\phi\pi^+\pi^-}^{(s)} \simeq -[5.6 \pm 0.4 - i(3.8 \pm 0.12)]10^{-3}.$$

Let us note that the error is mainly due to the differences between the L_i correspond-

ing to the different fits, since they are much larger than the errors given by MINUIT, which are certainly underestimated. Furthermore, we have checked that this error band spans the dispersion in the results due to the variations of the chiral parameters that could yield a reasonable fit.

Although they were not present in eq.(30) there are corrections coming from diagrams with photon loops which are expected to be of the same order of magnitude as the isospin breaking corrections from the different mass of charged and neutral mesons [12–15]. We do not have means at present to evaluate these diagrams within the non-perturbative chiral scheme which we have followed. One would also need counterterms whose values are nowadays unknown. However, explicit calculations of the absorptive part of the $\eta\gamma$ intermediate channel in ref. [7] give a contribution of, *at most*, 1/4 of the kaon loops but *with opposite sign*. This $\eta\gamma$ will be our largest source of uncertainty in the errors given for each one of the different $\phi - \omega$ scenarios, that we discuss next.

As we have already commented, the contribution from the two step $\phi - \omega - \rho$ mechanism, depends on the $\phi - \omega$ mixing. Our results are the following:

- Strong scenario: we find $g_{\phi\pi^+\pi^-}^{\omega\phi} = [4.4 - i3.7]10^{-3}$ or $g_{\phi\pi^+\pi^-}^{\omega\phi} = [6.0 - i5.6]10^{-3}$, depending on whether we use $\text{Re}\tilde{\Theta}_{\phi\omega} = 20000$ or 29000 MeV^2 , respectively. Therefore, there is a large cancellation with the kaon loop contribution, and we obtain:

$$BR \simeq (1.7 \pm 0.3) \times 10^{-4} \text{ to } (2.5 \pm 0.3) \times 10^{-4},$$

where the uncertainty in the central values depends on whether we use $\text{Re}\tilde{\Theta}_{\phi\omega} = 20000$ or 29000 MeV^2 , respectively.

- Weak scenario: we get $g_{\phi\pi^+\pi^-}^{\omega\phi} = [-0.73 - i0.61]10^{-3}$, very small compared with both the electromagnetic and kaon-loop contributions. Thus, there is only a partial cancellation of the electromagnetic contribution with that of kaon loops, and we obtain

$$BR \simeq (0.38 \pm 0.12) \times 10^{-4}.$$

Apart from the contributions discussed so far, there is also the possibility of local terms giving rise to a direct $\rho - \phi$ mixing. However, one can argue that, by resonance saturation, the inclusion of the two-step process $\phi - \omega - \rho$ can be enough to take care of such local terms by considering that they are resummed on the ω propagator.

V. CONCLUSIONS

In this work we have evaluated the kaon loop contribution to the $\phi \rightarrow \pi^+\pi^-$ decay via $\phi - \rho$ mixing from the splitting of meson masses, making use of the unitarized chiral amplitudes with strong isospin breaking. We have shown that although this strong contribution to the $\phi \rightarrow \pi^+\pi^-$ decay gives rise to smaller branching ratios by itself than the tree level electromagnetic contributions, they can have a very large destructive interference with either the electromagnetic or the $\phi - \omega - \rho$ contributions.

We have also estimated the error in our $\phi \rightarrow \pi^+\pi^-$ branching ratio calculation coming from the uncertainties in F_V , the fitted $\mathcal{O}(p^4)$ ChPT counterterms, the photon-loop contributions, as well as the considered $\phi - \omega$ mixing scenarios.

A complete calculation of the loops with photons is missing in the present work, although they have been estimated making use of the results of ref. [7]. Still, they are the main source of uncertainty within each $\phi - \omega$ mixing scenario.

Accepting this additional uncertainty, we find that the strong coupling scenario [8]

$$BR \simeq (1.7 \pm 0.3) \times 10^{-4} \text{ to } (2.5 \pm 0.3) \times 10^{-4},$$

in very good agreement with the experimental results of ref. [5]. In contrast, the Weak [8] scenario yields

$$BR \simeq (0.38 \pm 0.12) \times 10^{-4},$$

It seems to prefer a value somewhat lower than the experimental value provided by ref. [6], although still reasonably compatible with it.

Of course, a precise determination of the photon loops in the non-perturbative regime would be desirable to reduce the theoretical uncertainties.

Finally, we would like to remark that the solution of the experimental conflict in the $\phi \rightarrow \pi^+\pi^-$ will, eventually, help us to discard some of the $\phi-\omega$ mixing scenarios proposed in the literature.

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APPENDIX A: AMPLITUDES

In this appendix we give the expression for the $J = 1$ partial waves obtained from the ChPT Lagrangian, but setting $m_u \neq m_d$. The normalization of the T -matrix used here is the same as in ref. [17]. Let us first define the modulus of the CM momenta of the different particles as

$$p_{\pi^+} = \sqrt{\frac{s}{4} - m_{\pi^+}^2}, p_{K^+} = \sqrt{\frac{s}{4} - m_{K^+}^2}, p_{K^0} = \sqrt{\frac{s}{4} - m_{K^0}^2}.$$

where m_{π^+} is the charged pion mass. Then, once they are projected in P -wave, the tree level amplitudes from the $O(p^2)$ and $O(p^4)$ Lagrangian for $K^+K^- \rightarrow \pi^+\pi^-$ scattering are

$$T_2(s, t, u) = -\frac{p_{\pi^+} p_{K^+}}{3 f_{K^+} f_{\pi}},$$

$$T_4^P(s, t, u) = \frac{4}{3 f_{K^+}^2 f_{\pi}^2} [L_3 s - L_5(m_{K^+}^2 + m_{\pi^+}^2)]$$

whereas for $K^0\bar{K}^0 \rightarrow \pi^+\pi^-$ scattering they are given by

$$T_2(s, t, u) = \frac{p_{\pi^+} p_{K^0}}{3 f_{K^0} f_{\pi}},$$

$$T_4^P(s, t, u) = -\frac{4}{3 f_{K^0}^2 f_{\pi}^2} [L_3 s - L_5(m_{K^0}^2 + m_{\pi^+}^2)]$$

In the above formulas, f_{π} , $f_{K^+} = 1.22 f_{\pi}$ and f_{K^0} are the decay constants of the charged pion, kaon and neutral kaon, respectively. In the approach we are following here of neglecting tadpoles one has, up to $\mathcal{O}(p^4)$, that

$$f_{K^0} = f_{K^+} (1 + 4L_5 \frac{m_{K^0}^2 - m_{K^+}^2}{f_{\pi}^2}).$$

For $K^+K^- \rightarrow K^+K^-$ we obtain

$$T_2(s, t, u) = -\frac{2}{3 f_{K^+}^2} p_{K^+}^2,$$

$$T_4^P(s, t, u) = \frac{4 p_{K^+}^2}{3 f_{K^+}^4} [2(2L_1 - L_2 + L_3)s - 4(2L_4 + L_5)m_{K^+}^2],$$

the $K^0\bar{K}^0 \rightarrow K^0\bar{K}^0$ amplitude is exactly the same, but changing m_{K^+} by m_{K^0} and f_{K^+} by f_{K^0} . For $\pi^+\pi^- \rightarrow \pi^+\pi^-$, we find

$$T_2(s, t, u) = -\frac{2}{3 f_{\pi}^2} p_{\pi^+}^2,$$

$$T_4^P(s, t, u) = \frac{8 p_{\pi^+}^2}{3 f_{\pi}^4} [(2L_1 - L_2 + L_3)s - (4L_4 + 2L_5)m_{\pi^+}^2].$$

We have left the $K^+K^- \rightarrow K^0\bar{K}^0$ amplitude for the end, since we had an erratum in our previous paper [17]. Thus, we first give the *complete amplitude in the isospin limit*, before projecting on the P -wave. It reads

$$T_2(s, t, u) = \frac{u - 2m_K^2}{2f_K^2},$$

$$T_4(s, t, u) = \frac{-2}{f_K^4} [(4L_1 + L_3)(s - 2m_K^2)^2 + 2L_2(u - 2m_K^2)^2 + (2L_2 + L_3)(t - 2m_K^2)^2 + 8m_K^4(L_8 + 2L_6) - 2um_K^2L_5 - 8m_K^2(2m_K^2 - s)L_4].$$

The P -wave in the isospin breaking case is given by:

$$T_2(s, t, u) = -\frac{p_{K^+} p_{K^0}}{3 f_{K^+} f_{K^0}},$$

$$T_4^P(s, t, u) = \frac{4 p_{K^+} p_{K^0}}{3 f_{K^+}^2 f_{K^0}^2} [L_3 s - L_5(m_{K^+}^2 + m_{K^0}^2)].$$

Finally, we give the loop function $G = \text{diag}(g_1, g_2, g_3)$, where g_i is

$$g_i(s) = \frac{1}{(4\pi)^2} \left[\sigma_i \log \frac{\sigma_i Q_i + 1}{\sigma_i Q_i - 1} - 2 \log \left(\frac{q_{max}}{m_i} (1 + Q_i) \right) \right].$$

where $\sigma_i(s) = \sqrt{1 - 4m_i^2/s}$ and $Q_i = \sqrt{1 + m_i^2/q_{max}^2}$.

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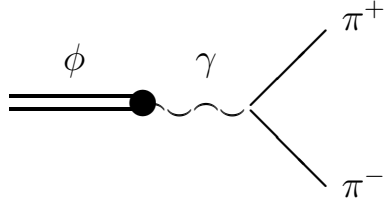


FIG. 1. $\phi \rightarrow \pi^+\pi^-$ decay through a photon.

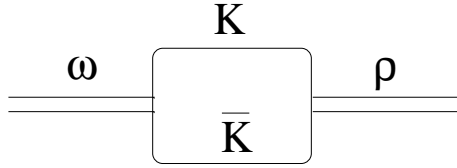


FIG. 2. Kaon loop contribution to the $\rho - \omega$ mixing.

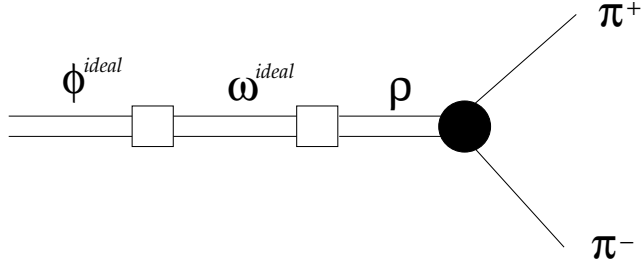


FIG. 3. Two step mechanism for $\phi \rightarrow \pi^+\pi^-$ decay.

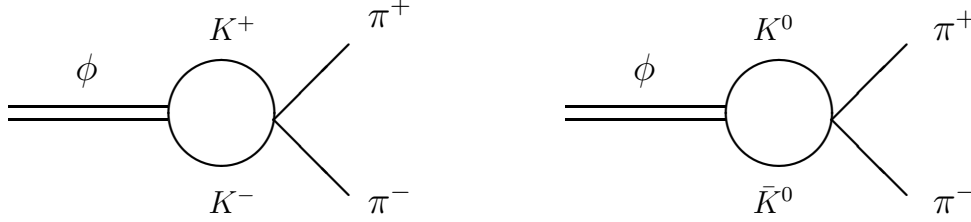


FIG. 4. Kaon loop contributions to the $\phi \rightarrow \pi^+\pi^-$ decay. If the charged and neutral kaons had the same mass, the two diagrams would cancel.

Fit	\hat{L}_1	\hat{L}_2	\hat{L}_3	\hat{L}_4	\hat{L}_5	$2\hat{L}_6 + \hat{L}_8$	\hat{L}_7	q_{max} (MeV)
set 1	0.91	1.61	-3.65	-0.25	1.07	0.58	-0.4	666
set 2	0.91	1.61	-3.65	-0.25	1.07	0.58	0.05	751
set 3	0.88	1.54	-3.66	-0.27	1.09	0.68	0.10	673
	L_1	L_2	L_3	L_4	L_5	$2L_6 + L_8$	L_7	μ
ChPT	0.4	1.4	-3.5	-0.3	1.4	0.5	-0.4	M_ρ
[31]	± 0.3	± 0.3	± 1.1	± 0.5	± 0.5	± 0.3	± 0.2	

TABLE I.

Different sets of chiral parameters (in 10^{-3} units) that yield reasonable fits to the meson-meson scattering phase shifts. We have used a hat to differentiate them from those obtained within standard ChPT [31], since in our case we have already differences at the $\mathcal{O}(p^4)$ with respect the next-to-leading ChPT amplitudes and we have used high energy data in the fit. However, as it is explained in the text, we still expect them to be relatively similar once the scales are chosen appropriately (roughly $\mu \simeq 1.2 q_{max}$, see [17] for details).

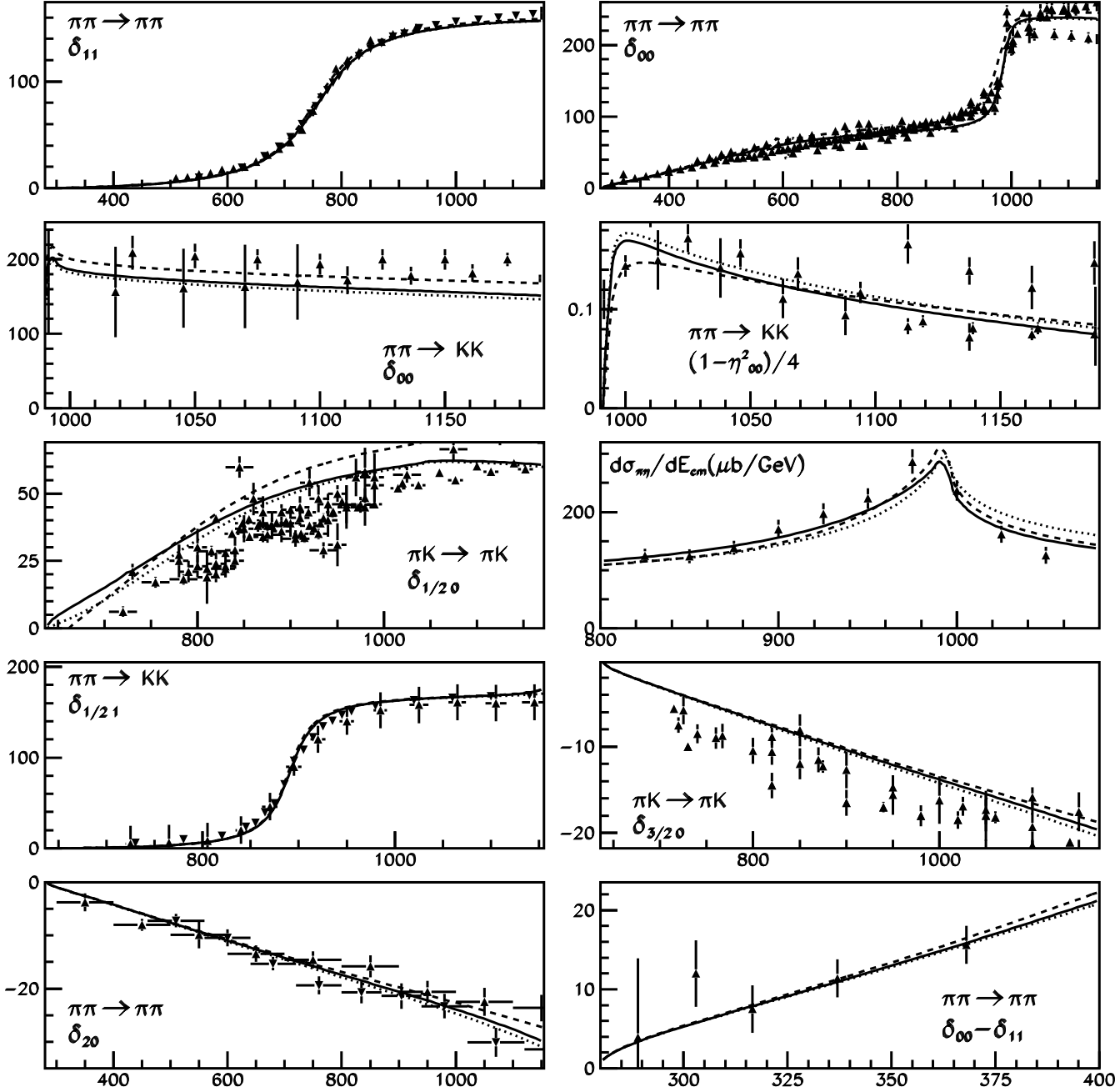


FIG. 5.

Coupled channel IAM results for meson-meson scattering. The dashed, continuous and dotted lines are obtained, respectively, with the chiral parameter sets 1, 2 and 3 given in table 1. Note that they are indistinguishable for almost every channel. The experimental data for each plot, starting from left to right and top to bottom, comes from [32,33], [32,34–36], [37,38], [35,37,38], [39–41], [42], [39,41], [41,43], [44,45] and, finally [29]

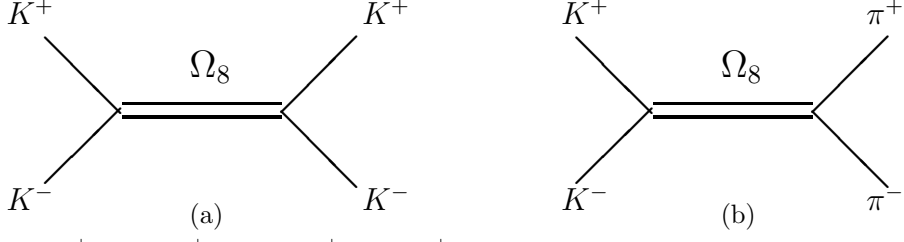


FIG. 6. $K^+K^- \rightarrow K^+K^-$ and $K^+K^- \rightarrow \pi^+\pi^-$ processes occurring through the exchange of an Ω_8 .

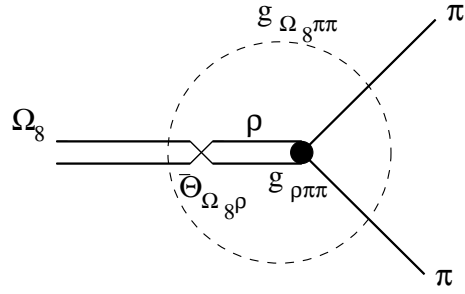


FIG. 7. The $g_{\Omega_8\pi^+\pi^-}$ coupling interpreted as a $\Omega_8 - \rho$ mixing and a $\rho \rightarrow \pi^+\pi^-$ decay.